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CELLULAR AUTOMATA

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CONTENTS

Index

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Cellular automata as models of complexity

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Natural systems from snowflakes to mollusc shells show a great diversity of complex patterns. The origins of such complexity can be investigated through mathematical models termed 'cellular automata'. Cellular automata consist of many identical components, each simple, but together capable of complex behaviour. They are analysed both as discrete dynamical systems, and as information-processing systems. Here some of their universal features are discussed, and some general principles are suggested.

IT is common in nature to find systems whose overall behaviour is extremely complex, yet whose fundamental component parts are each very simple. The complexity is generated by the cooperative effect of many simple identical components. Much has been discovered about the nature of the components in physical and biological systems; little is known about the mechanisms by which these components act together to give the overall complexity observed. What is now needed is a general mathematical theory to describe the nature and generation of complexity.

Cellular automata are examples of mathematical systems constructed from many identical components, each simple, but together capable of complex behaviour. From their analysis, one may, on the one hand, develop specific models for particular systems, and, on the other hand, hope to abstract general principles applicable to a wide variety of complex systems. Some recent results on cellular automata will now be outlined; more extensive accounts and references may be found in refs 1–4.

Cellular automata

A one-dimensional cellular automaton consists of a line of sites, with each site carrying a value 0 or 1 (or in general $0, \ldots, k-1$). The value a_i of the site at each position i is updated in discrete time steps according to an identical deterministic rule depending on a neighbourhood of sites around it:

$$a_i^{(t+1)} = \phi[a_{i-r}^{(t)}, a_{i-r+1}^{(t)}, \ldots, a_{i+r}^{(t)}] \qquad (1)$$

Even with $k = 2$ and $r = 1$ or 2, the overall behaviour of cellular automata constructed in this simple way can be extremely complex.

Consider first the patterns generated by cellular automata evolving from simple 'seeds' consisting of a few non-zero sites. Some local rules ϕ give rise to simple behaviour; others produce complicated patterns. An extensive empirical study suggests that the patterns take on four qualitative forms, illustrated in Fig. 1:

(1) disappears with time;
(2) evolves to a fixed finite size;
(3) grows indefinitely at a fixed speed;
(4) grows and contracts irregularly.

Patterns of type 3 are often found to be self-similar or scale invariant. Parts of such patterns, when magnified, are indistinguishable from the whole. The patterns are characterized by a fractal dimension[5]; the value $\log_2 3 \simeq 1.59$ is the most common. Many of the self-similar patterns seen in natural systems may, in fact, be generated by cellular automaton evolution.

Figure 3 shows the evolution of cellular automata from initial states where each site is assigned each of its k possible values with an independent equal probability. Self-organization is seen: ordered structure is generated from these disordered initial states, and in some cases considerable complexity is evident.

Different initial states with a particular cellular automaton rule yield patterns that differ in detail, but are similar in form and statistical properties. Different cellular automaton rules yield very different patterns. An empirical study, nevertheless, suggests that four qualitative classes may be identified, yielding four characteristic limiting forms:

(1) spatially homogeneous state;
(2) sequence of simple stable or periodic structures;
(3) chaotic aperiodic behaviour;
(4) complicated localized structures, some propagating.

All cellular automata within each class, regardless of the details of their construction and evolution rules, exhibit qualitatively similar behaviour. Such universality should make general results on these classes applicable to a wide variety of systems modelled by cellular automata.

Applications

Current mathematical models of natural systems are usually based on differential equations which describe the smooth variation of one parameter as a function of a few others. Cellular automata provide alternative and in some respects complemen-

Fig. 1 Classes of patterns generated by the evolution of cellular automata from simple 'seeds'. Successive rows correspond to successive time steps in the cellular automaton evolution. Each site is updated at each time step according to equation (1) by cellular automaton rules that depend on the values of a neighbourhood of sites at the previous time step. Sites with values 0 and 1 are represented by white and black squares, respectively. Despite the simplicity of their construction, patterns of some complexity are seen to be generated. The rules shown exemplify the four classes of behaviour found. (The first three are $k = 2$, $r = 1$ rules with rule numbers[1] 128, 4 and 126, respectively; the fourth is a $k = 2$, $r = 2$ rule with totalistic code[2] 52.) In the third case, a self similar pattern is formed.
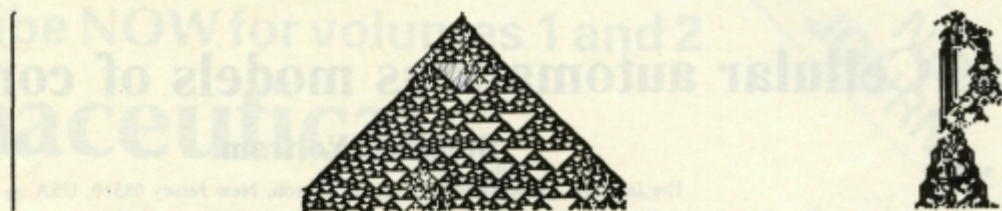
Fig. 2 Evolution of small initial perturbations in cellular automata, as shown by the difference (modulo two) between patterns generated from two disordered initial states differing in the value of a single site. The examples shown illustrate the four classes of behaviour found. Information on changes in the initial state almost always propagates only a finite distance in the first two classes, but may propagate an arbitrary distance in the third and fourth classes.

tary models, describing the discrete evolution of many (identical) components. Models based on cellular automata are typically most appropriate in highly nonlinear regimes of physical systems, and in chemical and biological systems where discrete thresholds occur. Cellular automata are particularly suitable as models when growth inhibition effects are important.

As one example, cellular automata provide global models for the growth of dendritic crystals (such as snowflakes)[6]. Starting from a simple seed, sites with values representing the solid phase are aggregated according to a two-dimensional rule that accounts for the inhibition of growth near newly-aggregated sites, resulting in a fractal pattern of growth. Nonlinear chemical reaction–diffusion systems give another example[7,8]: a simple cellular automaton rule with growth inhibition captures the essential features of the usual partial differential equations, and reproduces the spatial patterns seen. Turbulent fluids may also potentially be modelled as cellular automata with local interactions between discrete vortices on lattice sites.

If probabilistic noise is added to the time evolution rule (1), then cellular automata may be identified as generalized Ising models[9,10]. Phase transitions may occur if ϕ retains some deterministic components, or in more than one dimension.

Cellular automata may serve as suitable models for a wide variety of biological systems. In particular, they may suggest mechanisms for biological pattern formation. For example, the patterns of pigmentation found on many mollusc shells bear a striking resemblance to patterns generated by class 2 and 3 cellular automata (see refs 11, 12), and cellular automaton models for the growth of some pigmentation patterns have been constructed[13].

Mathematical approaches

Rather than describing specific applications of cellular automata, this article concentrates on general mathematical features of their behaviour. Two complementary approaches provide characterizations of the four classes of behaviour seen in Fig. 3.

In the first approach[2], cellular automata are viewed as discrete dynamical systems (see ref. 14), or discrete idealizations of partial differential equations. The set of possible (infinite) configurations of a cellular automaton forms a Cantor set; cellular automaton evolution may be viewed as a continuous mapping on this Cantor set. Quantities such as entropies, dimensions and Lyapunov exponents may then be considered for cellular automata.

In the second approach[3], cellular automata are instead considered as information-processing systems (see ref. 15), or parallel-processing computers of simple construction. Information represented by the initial configuration is processed by the evolution of the cellular automaton. The results of this information processing may then be characterized in terms of the types of formal languages generated. (Note that the mechanisms for information processing in natural system appear to be much closer to those in cellular automata than in conventional serial-processing computers: cellular automata may, therefore, provide efficient media for practical simulations of many natural systems.)

Entropies and dimensions

Most cellular automaton rules have the important feature of irreversibility: several different configurations may evolve to a single configuration, and, with time, a contracting subset of all possible configurations appears. Starting from all possible initial configurations, the cellular automaton evolution may generate only special 'organized' configurations, and 'self-organization' may occur.

For class 1 cellular automata, essentially all initial configurations evolve to a single final configuration, analogous to a limit point in a continuous dynamical system. Class 2 cellular automata evolve to limit sets containing essentially only periodic configurations, analogous to limit cycles. Class 3 cellular automata yield chaotic aperiodic limit sets, containing analogues of chaotic or 'strange' attractors.

Entropies and dimensions give a generalized measure of the density of the configurations generated by cellular automaton evolution. The (set) dimension or limiting (topological) entropy for a set of cellular automaton configurations is defined as (compare ref. 14)

$$d^{(x)} = \lim_{X \to \infty} \frac{1}{X} \log_k N(X) \tag{2}$$

where $N(X)$ gives the number of distinct sequences of X site values that appear. For the set of possible initial configurations, $d^{(x)} = 1$. For a limit set containing only a finite total number of configurations, $d^{(x)} = 0$. For most class 3 cellular automata, $d^{(x)}$ decreases with time, giving, $0 < d^{(x)} < 1$, and suggesting that a fractal subset of all possible configurations occurs.

A dimension or limiting entropy $d^{(t)}$ corresponding to the time series of values of a single site may be defined in analogy with equation (2). (The analogue of equation (2) for a sufficiently wide patch of sites yields a topologically-invariant entropy for the cellular automaton mapping.) $d^{(t)} = 0$ for periodic sets of configurations.

$d^{(x)}$ and $d^{(t)}$ may be modified to account for the probabilities of configurations by defining

$$d_\mu^{(x)} = -\lim_{X \to \infty} \frac{1}{X} \sum_{j=1}^{k^x} p_j \log_k p_j \tag{3}$$

and its analogue, where p_j are probabilities for possible length X sequences. These measure dimensions may be used to delineate the large time behaviour of the different classes of cellular automata:

(1) $d_\mu^{(x)} = d_\mu^{(t)} = 0$
(2) $d_\mu^{(x)} > 0, \ d_\mu^{(t)} = 0$
(3) $d_\mu^{(x)} > 0, \ d_\mu^{(t)} > 0$

As discussed below, dimensions are usually undefined for class 4 cellular automata.

Information propagation

Cellular automata may also be characterized by the stability or predictability of their behaviour under small perturbations in initial configurations. Figure 2 shows differences in patterns generated by cellular automata resulting from a change in a
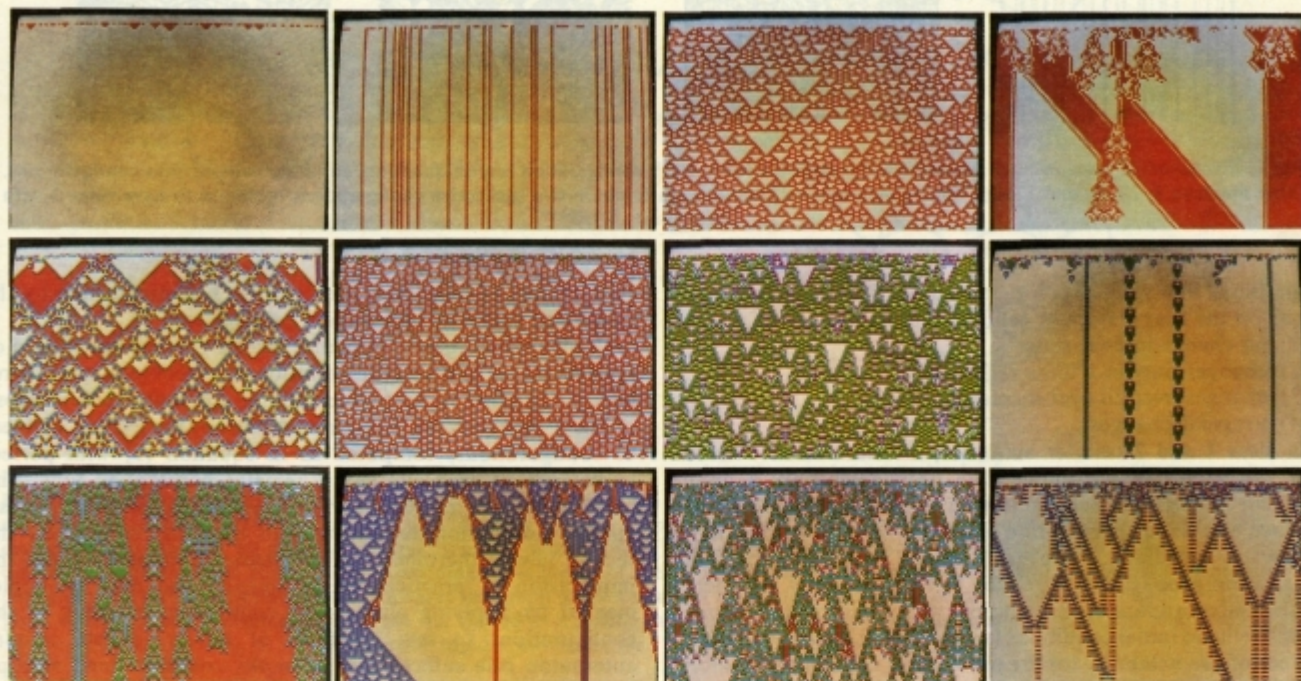
Fig. 3 Evolution of various cellular automata from disordered initial states. In many cases, ordered structure is seen to be generated. The first row of pictures show examples of the four qualitative classes of behaviour found. (The rules shown are the same as in Fig. 1.) The lower two rows show examples of cellular automata with $k = 5$ (five possible values for each site) and $r = 1$ (nearest neighbour rules). Site values 0 to 4 are represented by white, red, green, blue and yellow squares, respectively. (The rules shown have totalistic codes 10175, 566780, 570090, 580020, 583330, 672900, 5694390, 59123000.) The 'orange' discoloration is a background, not part of the pattern.

Fig. 4 Evolution of multiple phases in cellular automata. Pairs of sites are shown combined: 00 is represented by white, 01 by red, 10 by green and 11 by blue. Alternate time steps are shown. Both rules simulate an additive rule (number 90) under a blocking transformation. In the first rule (number 18), the simulation is attractive: starting from a disordered initial state, the domains grow with time. In the second rule (number 94), the simulation is repulsive: only evolution from a special initial state yields additive rule behaviour; a defect is seen to grow, and attractive simulation of the identity rule takes over.

Fig. 5 Examples of the evolution of a typical class 4 cellular automaton from disordered initial states. This and other class 4 cellular automata are conjectured to be capable of arbitrary information processing, or universal computation. The rule shown has $k = 3$, $r = 1$, and takes the value of a site to be 1 if the sum of the values of the sites in its three-site neighbourhood is 2 or 6, to be 2 if the sum is 3, and to zero otherwise (totalistic code 792).

Fig. 6 Persistent structures generated in the evolution of the class 4 cellular automaton of Fig. 5. The first four structures shown have periods 1, 20, 16 and 12 respectively; the last four structures (and their reflections) propagate: the first has period 32, the next three period 3, and the last period 6. These structures are some of the elements required to support universal computation.
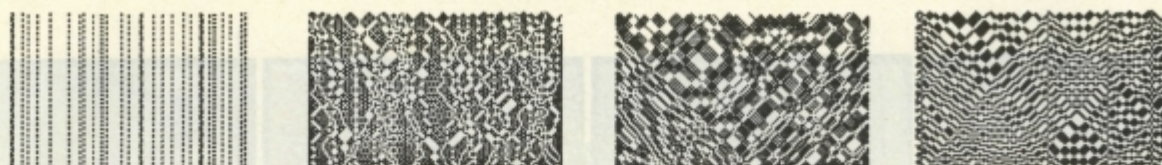
Fig. 7 Evolution of some cellular automata with reverisble rules. Each configuration is a unique function of the two previous configurations. (Rule numbers 4, 22, 90 and 126 are shown.) As initial conditions, each site in two successive configurations is chosen to have value 1 with probability 0.1.

single initial site value. Such perturbations have characteristic effects on the four classes of cellular automata:

   (1) no change in final state;
   (2) changes only in a finite region;
   (3) changes over an ever-increasing region;
   (4) irregular changes.

In class 1 and 2 cellular automata, information associated with site values in the initial state propagates only a finite distance; in class 3 cellular automata, it propagates an infinite distance at a fixed speed, while in class 4 cellular automata, it propagates irregularly, but over an infinite range. The speed of information propagation is related to the Lyapunov exponent for the cellular automaton evolution, and measures the degree of sensitivity to initial conditions (see ref. 16). It leads to different degrees of predictability for the outcome of cellular automaton evolution:

   (1) entirely predictable, independent of initial state;
   (2) local behaviour predictable from local initial state;
   (3) behaviour depends on an ever-increasing initial region;
   (4) behaviour effectively unpredictable.

Information propagation is particularly simple for the special class of additive cellular automata (whose local rule function ϕ is linear modulo k), in which patterns generated from arbitrary initial states may be obtained by superposition of patterns generated by evolution of simple initial states containing a single non-zero site. A rather complete algebraic analysis of such cellular automata may be given[17]. Most cellular automata are not additive; however, with special initial configurations it is often possible for them to behave just like additive rules. Thus, for example, the evolution of an initial configuration consisting of a sequence of 00 and 01 digrams under one rule may be identical to the evolution of the corresponding 'blocked' configuration consisting of 0 and 1 under another rule. In this way, one rule may simulate another under a blocking transformation (analogous to a renormalization group transformation). Evolution from an arbitrary initial state may be attracted to (or repelled from) the special set of configurations for which such a simulation occurs. Often several phases exist, corresponding to different blocking transformations: sometimes phase boundaries move at constant speed, and one phase rapidly takes over; in other cases, phase boundaries execute random walks, annihilating in pairs, and leading to a slow increase in the average domain size, as illustrated in Fig. 4. Many rules appear to follow attractive simulation paths to additive rules, which correspond to fixed points of blocking transformations, and thus exhibit self similarity. The behaviour of many rules at large times, and on large spatial scales, is therefore determined by the behaviour of additive rules.

Thermodynamics

Decreases with time in the spatial entropies and dimensions of equations (2) and (3) signal irreversibility in cellular automaton evolution. Some cellular automaton rules are, however, reversible, so that each and every configuration has a unique predecessor in the evolution, and the spatial entropy and dimension of equations (2) and (3) remain constant with time. Figure 7 shows some examples of the evolution of such rules, constructed by adding a term $-a_i^{(t-1)}$ to equation (1) (ref. 20 and E. Fredkin, personal communication). Again, there are analogues of the

four classes of behaviour seen in Fig. 3, distinguished by the range and speed of information propagation.

Conventional thermodynamics gives a general description of systems whose microscopic evolution is reversible; it may, therefore, be applied to reversible cellular automata such as those of Fig. 4. As usual, the 'fine-grained' entropy for sets (ensembles) of configurations, computed as in equation (3) with perfect knowledge of each site value, remains constant in time. The 'coarse-grained' entropy for configurations is, nevertheless, almost always non-decreasing with time, as required by the second law of thermodynamics. Coarse graining emulates the imprecision of practical measurements, and may be implemented by applying almost any contractive mapping to the configurations (a few iterations of an irreversible cellular automaton rule suffice). For example, coarse-grained entropy might be computed by applying equation (3) to every fifth site value. In an ensemble with low coarse-grained entropy, the values of every fifth site would be highly constrained, but arbitrary values for the intervening sites would be allowed. Then in the evolution of a class 3 or 4 cellular automaton the disorder of the intervening site values would 'mix' with the fifth-site values, and the coarse-grained entropy would tend towards its maximum value. Signs of self-organization in such systems must be sought in temporal correlations, often manifest in 'fluctuations' or metastable 'pockets' of order.

While all fundamental physical laws appear to be reversible, macroscopic systems often behave irreversibly, and are appropriately described by irreversible laws. Thus, for example, although the microscopic molecular dynamics of fluids is reversible, the relevant macroscopic velocity field obeys the irreversible Navier–Stokes equations. Conventional thermodynamics does not apply to such intrinsically irreversible systems: new general principles must be found. Thus, for cellular automata with irreversible evolution rules, coarse-grained entropy typically increases for a short time, but then decreases to follow the fine-grained entropy. Measures of the structure generated by self-organization in the large time limit are usually affected very little by coarse graining.

Formal language theory

Quantities such as entropy and dimension, suggested by information theory, give only rough characterizations of cellular automaton behaviour. Computation theory suggests more complete descriptions of self-organization in cellular automata (and other systems). Sets of cellular automaton configurations may be viewed as formal languages, consisting of sequences of symbols (site values) forming words according to definite grammatical rules.

The set of all possible initial configurations corresponds to a trivial formal language. The set of configurations obtained after any finite number of time steps are found to form a regular language[3]. The words in a regular language correspond to the possible paths through a finite graph representing a finite state machine. It can be shown that a unique smallest finite graph reproduces any given regular language (see ref. 15). Examples of such graphs are shown in Fig. 8. These graphs give complete specifications for sets of cellular automaton configurations (ignoring probabilities). The number of nodes Ξ in the smallest graph corresponding to a particular set of configurations may
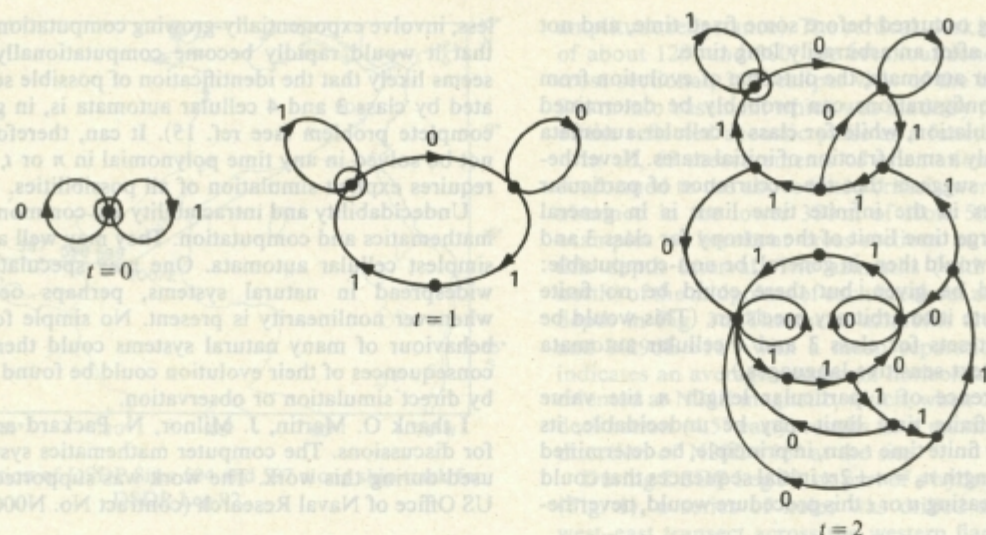
Fig. 8 Graphs representing the sets of configurations generated in the first few time steps of evolution according to a typical class 3 cellular automaton rule ($k = 2$, $r = 1$, rule number 126). Possible configurations correspond to possible paths through the graphs, beginning at the encircled node. At $t = 0$, all possible configurations are allowed. With time, a contracting subset of configurations are generated. (After one time step, for example, no configuration containing the sequence of site value 101 can appear.) At each time step, the complete set of possible configurations forms a regular formal language: the graph gives a minimal complete specification of it. The number of nodes in the graph gives a measure of the complexity Ξ of the set, viewed as a regular language. As for other class 3 cellular automata, the complexity of the sets grows rapidly with time; for $t = 3$, $\Xi = 107$, and $t = 4$, $\Xi = 2,867$.

be defined as the 'regular language complexity' of the set. It specifies the size of the minimal description of the set in terms of regular languages. Larger Ξ correspond to more complicated sets. (Note that the topological entropy of a set is given by the logarithm of the algebraic integer obtained as the largest root of the characteristic polynomial for the incidence matrix of the corresponding graph. The characteristic polynomials for the graphs in Fig. 7 are $2 - \lambda$ ($\lambda_{max} = 2$), $1 - \lambda + 2\lambda^2 - \lambda^3$ ($\lambda_{max} = 1.755$) and $-1 + \lambda - \lambda^2 + 2\lambda^3 - 4\lambda^4 + \lambda^5 + 3\lambda^6 - 5\lambda^7 + 3\lambda^8 - 3\lambda^9 + 5\lambda^{10} - 6\lambda^{11} + 4\lambda^{12} - \lambda^{13}$ ($\lambda_{max} \simeq 1.732$), respectively.)

The regular language complexity Ξ for sets generated by cellular automaton evolution almost always seems to be non-decreasing with time. Increasing Ξ signals increasing self-organization. Ξ may thus represent a fundamental property of self-organizing systems, complementary to entropy. It may, in principle, be extracted from experimental data.

Cellular automata that exhibit only class 1 and 2 behaviour always appear to yields sets that correspond to regular languages in the large time limit. Class 3 and 4 behaviour typically gives rise, however, to a rapid increase of Ξ with time, presumably leading to limiting sets not described by regular languages.

Formal languages are recognized or generated by idealized computers with a 'central processing unit' containing a fixed finite number of internal states, together with a 'memory'. Four types of formal languages are conventionally identified, corresponding to four types of computer:

● Regular languages: no memory required.
● Context-free languages: memory arranged as a last-in, first-out stack.
● Context-sensitive languages: memory as large as input word required.
● Unrestricted languages: arbitrarily large memory required (general Turing machine).

Examples are known of cellular automata whose limiting sets correspond to all four types of language (L. Hurd, in preparation). Arguments can be given that the limit sets for class 3 cellular automata typically form context-sensitive languages, while those for class 4 cellular automata correspond to unrestricted languages. (Note that while a minimal specification for any regular language may always be found, there is no finite procedure to obtain a minimal form for more complicated formal languages: no generalization of the regular language complexity Ξ may thus be given.)

Computation theory

While dynamical systems theory concepts suffice to define class 1, 2 and 3 cellular automata, computation theory is apparently required for class 4 cellular automata. Examples of the evolution of a typical class 4 cellular automaton are shown in Fig. 5. Varied and complicated behaviour, involving many different time scales is evident. Persistent structures are often generated; the smallest few are illustrated in Fig. 6, and are seen to allow both storage and transmission of information. It seems that the structures supported by this and other class 4 cellular automata rule may be combined to implement arbitrary information processing operations. Class 4 cellular automata would then be capable of universal computation: with particular initial states, their evolution could implement any finite algorithm. (Universal computation has been proved for a $k = 18$, $r = 1$ rule[22], and for two-dimensional cellular automata such as the 'Game of Life'[22,23].) A few per cent of cellular automaton rules with $k > 2$ or $r > 1$ are found to exhibit class 4 behaviour: all these would then, in fact, be capable of arbitrarily complicated behaviour. This capability precludes a smooth infinite size limit for entropy or other quantities: as the size of cellular automaton considered increases, more and more complicated phenomena may appear.

Cellular automaton evolution may be viewed as a computation. Effective preidiction of the outcome of cellular automaton evolution requires a short-cut that allows a more efficient computation than the evolution itself. For class 1 and 2 cellular automata, such short cuts are clearly possible: simple computations suffice to predict their complete future. The computational capabilities of class 3 and 4 cellular automata may, however, be sufficiently great that, in general, they allow no short-cuts. The only effective way to determine their evolution from a given initial state would then be by explicit observation or simulation: no finite formulae for their general behaviour could be given. (If class 4 cellular automata are indeed capable of universal computation, then the variety of their possible behaviour would preclude general prediction, and make explicit observation or simulation necessary.) Their infinite time limiting behaviour could then not, in general, be determined by any finite computational process, and many of their limiting properties would be formally undecidable. Thus, for example, the 'halting problem' of determining whether a class 4 cellular automaton with a given finite initial configuration ever evolves to the null configuration would be undecidable. An explicit simulation could determine

only whether halting occurred before some fixed time, and not whether it occurred after an arbitrarily long time.

For class 4 cellular automata, the outcome of evolution from almost all initial configurations can probably be determined only by explicit simulation, while for class 3 cellular automata this is the case for only a small fraction of initial states. Nevertheless, this possibility suggests that the occurrence of particular site value sequences in the infinite time limit is in general undecidable. The large time limit of the entropy for class 3 and 4 cellular automata would then, in general, be non-computable: bounds on it could be given, but there could be no finite procedure to compute it to arbitrary precision. (This would be the case if the limit sets for class 3 and 4 cellular automata formed at least context-sensitive languages.)

While the occurrence of a particular length n site value sequence in the infinite time limit may be undecidable, its occurrence after any finite time t can, in principle, be determined by considering all length $n_0 = n + 2rt$ initial sequences that could evolve to it. For increasing n or t this procedure would, neverthe-

less, involve exponentially-growing computational resources, so that it would rapidly become computationally intractable. It seems likely that the identification of possible sequences generated by class 3 and 4 cellular automata is, in general, an NP-complete problem (see ref. 15). It can, therefore, presumably not be solved in any time polynomial in n or t, and essentially requires explicit simulation of all possibilities.

Undecidability and intractability are common in problems of mathematics and computation. They may well afflict all but the simplest cellular automata. One may speculate that they are widespread in natural systems, perhaps occurring almost whenever nonlinearity is present. No simple formulae for the behaviour of many natural systems could then be given; the consequences of their evolution could be found effectively only by direct simulation or observation.

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ARTICLES

Orientation of *in situ* stresses in the oceanic crust

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Two in situ *measurements of principal stress directions have been made in DSDP Holes 504B, south of the Costa Rica Rift on the Nazca plate, and 597C, west of the East Pacific Rise on the Pacific plate. In both cases, the orientations of* in situ *principal stresses determined from borehole breakouts are consistent with the stress directions inferred from intraplate earthquakes located near the sites.*

WELLBORE breakouts are intervals of borehole elongation which are caused by preferential spalling in a zone where the circumferential compressive stress is greatest[1], and in which the average azimuth of the long dimension is consistent in a given well or field[2,3]. Stress-induced wellbore breakouts form at an azimuth perpendicular to the maximum horizontal principal stress direction[1]. Wellbore breakouts have been reported in wells from several parts of North America[2-15] but there have been problems in their identification (see ref. 16). There now seems to be ample evidence that breakouts can be used as a reliable indicator of the orientation of the horizontal principal stresses. Here, we use specially processed borehole televiewer data to study the detailed shape of breakouts (see ref. 1).

Borehole televiewer reflectivity records have previously been used in the study of fractures intersecting wellbores at depth (see ref. 17), for identifying lithostratigraphical features in deep ocean boreholes (see refs 18, 19), and in identifying zones of borehole breakout[1,20,21].

Experimental data

Evidence of the orientation of the contemporary *in situ* stress field can be found at two DSDP (Deep Sea Drilling Project) sites in the oceanic crust (Fig. 1). DSDP Site 504 is located 200 km due south of the Costa Rica Rift and about 350 km north-west of the Peru–Chile Trench. DSDP Site 597 is located about 1,800 km west of the East Pacific Rise and 2,000 km east of Tahiti.

During DSDP Leg 83, the DV *Glomar Challenger* was drilling Hole 504B to a depth of 1,350 m, 1,000 m of which was into basement[22] (Fig. 1). The upper 100 m of basement[19] is composed mainly of pillow basalts and thin flow units of seismic layer 2A. The next 550 m of pillows, breccias and flows is a site of intense fracturing, and contains extensive secondary alteration; it is thought to be seismic layer 2B. The bottom 350 m of the hole is composed mainly of massive, 'welded' units, or sheeted dykes, and is the upper part of seismic layer 2C.